

Excluding the local hidden variable theory with time-reversal Bell test

Yong-gang Tan^{1,*}

¹*Physics and Information Engineering Department, Luoyang Normal College,
Luoyang 471022, Henan, People's Republic of China*

A time-reversal Bell test protocol is proposed. The quantum states are prepared by faraway separated partners and transferred to the third partner who carries out Bell basis measurement on them to post-select the Einstein-Podolsky-Rosen (EPR) pairs. If some loopholes open, similar as that in normal Bell test, the Bell violation in the present protocol is apt to be interpreted with local hidden variable (lhv) theory. With some modifications on the protocol, the lhvs at both sides are prevented to exchange their information. Thus they only function locally and cannot affect the behaviors of the states at the other sides. However, Bell violation can still be obtained in this case. It means that Bell violation is realized with the lhv theory excluded. Because high detection efficiency is not compulsory, this protocol can be realized with present technology.

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I. INTRODUCTION

In 1935, Einstein, Podolsky, and Rosen argued that the wavefunction description of quantum physics is incomplete unless lhv exists [1]. In 1964, Bell proposed the first experimentally realizable model to test whether lhv exists or not [2]. Presently, the most widely used Bell-type inequalities are the Clauser-Horne-Shimony-Holt (CHSH) inequality [3] and the Clauser-Horne (CH) inequality [4]. Unfortunately, there is no conclusive verdict on the correctness of the EPR's standpoint due to the loopholes existing in current Bell experiments [5–17]. Therefore, the realization of loophole-free Bell experiments becomes one of the most significant issues in fundamental physics.

Normally, the locality loophole exists in Bell test with massive particles, while the detection loophole is often present in Bell test with optical system. For Bell test carried out with optical systems, it has been shown that the detection loophole can be closed only when the detection efficiency η is higher than 82.8% [3, 4, 18], which is too high to be realized with current technology. To close the detection loophole, one should either improve the performances of the devices or decrease the requirements on the detection efficiency. For Bell test with CH inequality, no fair-sampling assumption is needed. If Bell violation is obtained, the lhv theory is excluded. When non-maximally entangled quantum states are used, it is proven that one can realize the Bell violation with the detection efficiency higher than 66.7% [19–21].

Because comparably lower detection efficiency can be used for Bell test with CH inequality to realize the Bell violation, it has attracted wide interesting in optical Bell experiment with present detection technology [22, 23]. In order to maximize the Bell violation in Bell test with CH inequality, however, special states and measurement directions should be chosen [19–21]. Furthermore, the tolerable noise is very low when inefficient detectors are used. To our knowledge, Bell test with all loopholes closed is still the aim pursued by scientists in the field of fundamental physics.

Recently, measurement-device independent quantum key distribution (MDI-QKD) protocol has been proposed, where the EPR pairs post-selected in the Bell basis measurement ensure the security of the protocol [24–26]. Entanglement is the essential nature of quantum physics, it does not matter whether the entanglement is generated by a parametric-down-conversion (PDC) source or post-selected in the Bell basis measurement. Thus the the entanglement generated in the latter way may also be used in the Bell experiment to test the lhv theory. In this letter, a time-reversal Bell test protocol is proposed, where EPR pairs post-selected in the Bell basis measurement are used. Our protocol can exclude the lhv theory with available detection technology presently.

*Electronic address: ygtan@lynu.edu.cn

II. TIME-REVERSAL BELL TEST

In Bell test with CHSH inequality, the lhv theory requires that

$$S_{\text{CHSH}} \equiv \langle A_1 B_1 + A_1 B_2 + A_2 B_1 - A_2 B_2 \rangle \leq 2. \quad (1)$$

Here $\langle M \rangle$ is the expected value of M , A_i , B_j are the observable variables chosen by Alice and by Bob, with $i, j \in \{1, 2\}$. Quantum-mechanically, the maximal value of S_{CHSH} is proven to be $2\sqrt{2}$ [27]. In practical condition where the quantum channels are lossy and the single-photon detectors are inefficient, partial photons are absorbed within the transmission process and finally missed by the detectors. If the detection efficiency is η at both sides, closing the detection loophole in Bell test with CHSH inequality means that [18]

$$\frac{2\sqrt{2}\eta^2}{\eta^2 + 2\eta(1-\eta)} > 2. \quad (2)$$

Thus $\eta > 82.8\%$ is required to realize detection loophole-free Bell test with CHSH inequality.

In Bell test with CH inequality, the lhv theory suggests

$$S_{\text{CH}} = P(a_1, b_1) + P(a_2, b_1) + P(a_1, b_2) - P(a_2, b_2) - P(a_1) - P(b_1) \leq 0, \quad (3)$$

where $P(a_i)$ and $P(b_j)$ are the probabilities that Alice and Bob have efficient detections in a_i and b_j , respectively. The most common bi-partite entanglement states used in the Bell experiments are the Bell states

$$\begin{aligned} |\Phi_{AB}^+\rangle &= \frac{1}{\sqrt{2}} (|00\rangle_{AB} + |11\rangle_{AB}), \\ |\Phi_{AB}^-\rangle &= \frac{1}{\sqrt{2}} (|00\rangle_{AB} - |11\rangle_{AB}), \\ |\Psi_{AB}^+\rangle &= \frac{1}{\sqrt{2}} (|01\rangle_{AB} + |10\rangle_{AB}), \\ |\Psi_{AB}^-\rangle &= \frac{1}{\sqrt{2}} (|01\rangle_{AB} - |10\rangle_{AB}). \end{aligned} \quad (4)$$

If inefficient detectors are used, however, the value of the CH polynomial is maximized by

$$\rho_{AB}^\theta = (1-p)|\Phi_{AB}^\theta\rangle\langle\Phi_{AB}^\theta| + p\frac{\mathbf{I}}{4}, \quad (5)$$

where $|\Phi_{AB}^\theta\rangle = \cos\theta|00\rangle_{AB} + \sin\theta|11\rangle_{AB}$, and \mathbf{I} is the identity matrix used to denote the white noise. The minimum detection efficiency to violate the CH inequality can be reduced to be 66.7% by choosing appropriate θ , p , a_i s and b_j s [19–21].

The MDI-QKD has found great interesting in quantum information as well as in fundamental physics fields. In the protocol, both Alice and Bob have well controlled state-preparing devices. They randomly choose their bases to prepare their quantum states on their photons which are then distributed to the third party, Eve, who may be dishonest and harmful to the security of the communication. Eve is required to implement Bell basis measurement on the incoming photons and publish her measurement outcomes. The MDI-QKD is proven to be a time-reversal entanglement-based QKD protocol [24–26]. The monogamy of entanglement can thus ensure the security of the key bits [28–32]. As entanglement is post-selected within Bell basis measurement, it may be used in Bell test to violate the Bell inequalities. The time-reversal Bell test is described as followings.

(a) As is shown in Fig.1 (a), Alice chooses her basis randomly to be A_1 and A_2 , and Bob randomly chooses his bases as B_1 and B_2 . They also randomly choose their bit values between -1 and 1 . The photons with well prepared states are transferred to their trust partner, Charlie, who is set between them.

(b) The photons are guided to coincide on the beam splitter (BS) and measured by a set of Bell basis measurement device. Charlie announces Alice and Bob his measurement outcomes. That is, which quantum state is obtained within the measurement, $|\Psi_{AB}^+\rangle$, $|\Psi_{AB}^-\rangle$, $|\Phi_{AB}^+\rangle$, or $|\Phi_{AB}^-\rangle$.

(c) Alice and Bob keep their basis choices and bit value choices when Charlie's measurement outcomes turn out to be $|\Phi_{AB}^+\rangle$ and $|\Psi_{AB}^-\rangle$. They use their basis choices and bit value choices to calculate the CHSH polynomial in Eq.(1).

The time-reversal version of the present protocol can be characterized as followings.

(a') Charlie is set between Alice and Bob. He randomly prepares his quantum states to be $|\Psi_{AB}^+\rangle$, $|\Psi_{AB}^-\rangle$, $|\Phi_{AB}^+\rangle$, and $|\Phi_{AB}^-\rangle$. Then he distributes Photon A to Alice and Photon B to Bob.

(b') Alice takes Photon A and Bob takes Photon B. Alice randomly chooses her measurement settings to be A_1 and A_2 , while Bob randomly selects his measurement settings to be B_1 and B_2 . They carry out projective measurements on the quantum states of their photons, respectively.

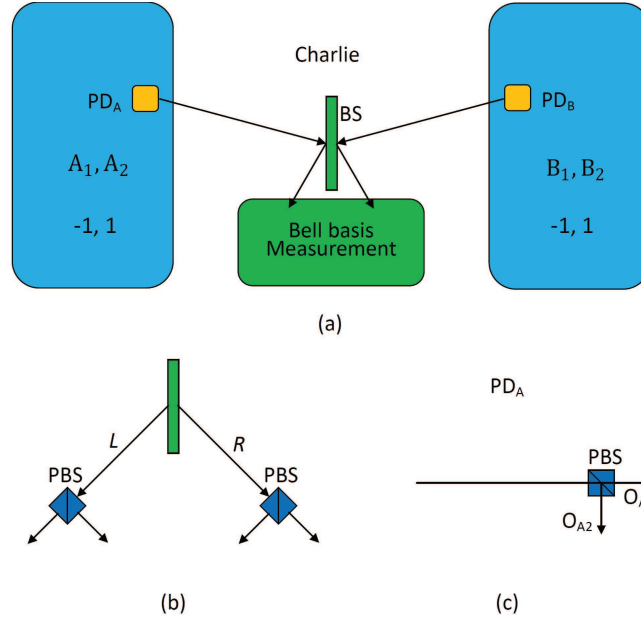


FIG. 1: (Color online) (a) Alice randomly prepares her photons with randomly chosen bases, A_1 and A_2 , and bit values, -1 and 1 . Similarly, Bob randomly chooses his bases between B_1 and B_2 , and bit values -1 and 1 to prepare his photons. The photons are transferred to Charlie who mixes the paths of the photons with a beam splitter (BS) and implements Bell basis measurement on them. (b) Scheme of partial Bell basis measurement with optical systems. It can be used to differentiate $|\Psi_{AB}^+\rangle$ from $|\Psi_{AB}^-\rangle$. The former happens when both Channel L and Channel R have outputs of different polarizations, while the latter occurs when either Channel L or Channel R has outputs of different polarizations. (c) Schematic of PD_A. Alice departs different polarization components of her photon with a polarization beam splitter (PBS).

(c') Alice and Bob publish their basis choices and measurement outcomes. At the same time, Charlie announces Alice and Bob which Bell state has been generated in his lab. They use the $|\Phi_{AB}^+\rangle$ and $|\Psi_{AB}^-\rangle$ events to calculate the CHSH polynomial.

Thus the present protocol is a time-reversal Bell test protocol. Quantum principle requires that the events measured to be $|\Phi_{AB}^+\rangle$ make the maximal value of S_{CHSH} reaches $2\sqrt{2}$, while the events measured to be $|\Psi_{AB}^-\rangle$ make the minimal value of S_{CHSH} reaches $-2\sqrt{2}$.

III. LHV MODEL OF THE TIME-REVERSAL BELL TEST

In the present protocol, Alice and Bob prepare their quantum states in their own labs. They choose their bases and bit values randomly with genuinely random number generators. If the time intervals between their state preparations are very short in every run, the lhv theory does not allow the basis choices and bit value choices at one side to be affected by those at the other side. Namely, $\langle A_i B_j \rangle = \langle A_i \rangle \langle B_j \rangle$ should be satisfied. One can thus obtain

$$S_{\text{CHSH}} = \langle A_1 \rangle \langle B_1 \rangle + \langle A_1 \rangle \langle B_2 \rangle + \langle A_2 \rangle \langle B_1 \rangle - \langle A_2 \rangle \langle B_2 \rangle. \quad (6)$$

It is easy to verify that $-2 \leq S_{\text{CHSH}} \leq 2$ when $-1 \leq \langle A_i \rangle \leq 1$ and $-1 \leq \langle B_j \rangle \leq 1$ are satisfied.

The detection loophole may be the biggest obstacle in excluding the lhv theory for Bell test with optical systems. In practical Bell test where inefficient detectors are used, partial photons are absorbed within the transmission process. Though Bell violation can be obtained for those events that both sides have efficient detections, no Bell violation is obtained when the unregistered events are included. It is because the values on the unregistered photons are uncertain. This opens the loophole for the unfair-sampling lhv. If the unfair-sampling lhv exists and decides the fates of the photon. It can only pick out the maximally violated events to be registered. If the lhv theory is correct, the events that both sides have clicks cannot violate the CHSH inequality on behalf of the whole. The unfair-sampling lhv should also be considered in the present protocol if inefficient detectors are used. When partial photons are missed within the transmission process, Charlie cannot make sure which Bell state should be assigned to the measurement outcomes. If no Bell violation can be obtained when these uncertain outcomes are incorporated, the registered $|\Phi_{AB}^+\rangle$ or $|\Psi_{AB}^-\rangle$ events cannot be used to violate the CHSH inequality on behalf of the whole.

Suppose that the detection efficiency at both sides is η . Because Alice and Bob randomly choose their bit values in every state-preparing basis, $|\Phi_{AB}^+\rangle$, $|\Phi_{AB}^-\rangle$, $|\Psi_{AB}^+\rangle$ and $|\Psi_{AB}^-\rangle$ happen equiprobably in the Bell basis measurement. Charlie cannot decide which state is generated in his Bell basis measurement until both sides have efficient pulses injecting in his lab and finally detected by his detectors. With probability $\frac{\eta^2}{4}$, Charlie has a register of $|\Phi_{AB}^+\rangle$ in his Bell basis measurement. The probability Charlie fails to read out the measurement outcomes is $4 \times \frac{(1-\eta^2)}{4}$. From the time-reversal aspect, the measurement outcomes $|\Phi_{AB}^+\rangle$ mean that the corresponding states prepared by Alice and by Bob maximize S_{CHSH} to be $2\sqrt{2}$. Because the four Bell states are equiprobable, the value of S_{CHSH} for the uncertain measurement outcomes is averaged to be 0. To refute the unfair-sampling lhv, one obtains that

$$\frac{2\sqrt{2}\eta^2}{\eta^2 + 4(1 - \eta^2)} > 2. \quad (7)$$

Thus $\eta > 95.19\%$ is required to excluding the unfair-sampling lhv. It is to higher to be realized with present technology.

In normal Bell experiments, Alice and Bob are required to randomly choose their measurement settings [17]. Or else, the deterministic lhv may exist [33]. Experimentally, faked Bell violation has been realized perfectly if Alice's and Bob's freedoms to choose their bases are not allowed [34]. In the present protocol, the basis choices and the bit value choices of Alice's and Bob's states are well chosen as soon as they are prepared. It means that their basis choices are determined when the photons are prepared even if Alice and Bob can choose the bases randomly. Thus deterministic lhv should also be considered in the present protocol.

IV. EXCLUDING THE LHV THEORY WITH MODIFIED TIME-REVERSAL BELL TEST

The lhv theory is introduced to make the description of quantum physics complete. If the lhv theory is correct, the lhvs should be essential attributes of the quantum world. In Bell test with optical systems, the lhvs are parasitic on the states of the photons. Thus the lhvs should have the same flying speed as that of the photons.

Suppose that the state encoded on Alice's photon is $\gamma_A|0\rangle_A + \delta_A|1\rangle_A$, and the state encoded on Bob's photon is $\gamma_B|0\rangle_B + \delta_B|1\rangle_B$. The joint state on the photons can thus be written as $\Gamma_{AB} = \gamma_A\gamma_B|0\rangle_A|0\rangle_B + \gamma_A\delta_B|0\rangle_A|1\rangle_B + \delta_A\gamma_B|1\rangle_A|0\rangle_B + \delta_A\delta_B|1\rangle_A|1\rangle_B$. According to the time-reversal formation of entanglement, after the BS, the part $\gamma_A\gamma_B|0\rangle_A|0\rangle_B + \delta_A\delta_B|1\rangle_A|1\rangle_B$ collapses into $|\Phi_{AB}^+\rangle$ and $|\Phi_{AB}^-\rangle$ within the Bell basis measurement. For the part $\gamma_A\delta_B|0\rangle_A|1\rangle_B + \delta_A\gamma_B|1\rangle_A|0\rangle_B$, however, it collapses into $|\Psi_{AB}^+\rangle$ and $|\Psi_{AB}^-\rangle$ in the Bell basis measurement after the BS.

Experimentally, it is still difficult to realize full Bell basis measurement. As is shown in Fig.1 (b), we consider the partial Bell basis measurement case where only $|\Psi_{AB}^+\rangle$ and $|\Psi_{AB}^-\rangle$ can be differentiated. With simple calculation, one can obtain

$$\begin{aligned} & \gamma_A\delta_B|0\rangle_A|1\rangle_B + \delta_A\gamma_B|1\rangle_A|0\rangle_B \\ \rightarrow & \frac{(\delta_A\gamma_B - \gamma_A\delta_B)}{\sqrt{2}}|\Psi_{AB}^-\rangle + \frac{i(\delta_A\gamma_B + \gamma_A\delta_B)}{\sqrt{2}}|\Psi_{AB}^+\rangle. \end{aligned} \quad (8)$$

The probabilities for $|\Psi_{AB}^+\rangle$ and $|\Psi_{AB}^-\rangle$ to occur are different, and their total probability is calculated to be $\gamma_A^2\delta_B^2 + \delta_A^2\gamma_B^2$.

It is impossible to filter out the part $\gamma_A\gamma_B|0\rangle_A|0\rangle_B + \delta_A\delta_B|1\rangle_A|1\rangle_B$ from the part $\gamma_A\delta_B|0\rangle_A|1\rangle_B + \delta_A\gamma_B|1\rangle_A|0\rangle_B$ without destroying Γ_{AB} . If measurements are carried out on Γ_{AB} in the rectilinear basis, four possible measurement outcomes occur, namely, $|0\rangle_A|0\rangle_B$, $|1\rangle_A|1\rangle_B$, $|0\rangle_A|1\rangle_B$, and $|1\rangle_A|0\rangle_B$. As is shown in Fig.1 (c), PD_A and PD_B are introduced to realize the measurements. The PD_A is composed with a polarization beam splitter (PBS) and several segments of quantum channels. The photon with well prepared quantum state is fed into the PBS whose outputs are then forwarded to Charlie. PD_B is operated by Bob in the same way.

If the bases of both PBSes are aligned with the rectilinear basis, after the PBSes, the joint state on Photon A and Photon B is $\gamma_A^2\gamma_B^2|00\rangle_{AB}\langle 00| + \gamma_A^2\delta_B^2|01\rangle_{AB}\langle 01| + \delta_A^2\gamma_B^2|10\rangle_{AB}\langle 10| + \delta_A^2\delta_B^2|11\rangle_{AB}\langle 11|$. The part $\gamma_A^2\delta_B^2|01\rangle_{AB}\langle 01| + \delta_A^2\gamma_B^2|10\rangle_{AB}\langle 10|$ is collapsed to be $|\Psi_{AB}^+\rangle$ and $|\Psi_{AB}^-\rangle$, while the left is collapsed to be $|\Phi_{AB}^+\rangle$ or $|\Phi_{AB}^-\rangle$ within the Bell basis measurement. Because that

$$\begin{aligned} |01\rangle_{AB} & \rightarrow \frac{1}{\sqrt{2}}(i|\Psi^+\rangle_{AB} + |\Psi^-\rangle_{AB}), \\ |10\rangle_{AB} & \rightarrow \frac{1}{\sqrt{2}}(i|\Psi^+\rangle_{AB} - |\Psi^-\rangle_{AB}), \end{aligned} \quad (9)$$

$|\Psi^+\rangle_{AB}$ and $|\Psi^-\rangle_{AB}$ happen with equal probability $\frac{\gamma_A^2\delta_B^2 + \delta_A^2\gamma_B^2}{2}$. The total probability of these two events is calculated to be $\gamma_A^2\delta_B^2 + \delta_A^2\gamma_B^2$, which is the same as that when no PBS is arranged in the experiment. Compared (9) with (8),

though the PBSes destroy the original quantum states of Alice and Bob, the anti-correlated components of Alice's and Bob's quantum states are completely picked out to be measured as $|\Psi_{AB}^+\rangle$ or $|\Psi_{AB}^-\rangle$ in both cases. The total probabilities of $|\Psi_{AB}^+\rangle$ and $|\Psi_{AB}^-\rangle$ are calculated to be the same in both cases.

Alice's and Bob's states can be formally rewritten as $\frac{(\gamma_A+\delta_A)}{\sqrt{2}}|+\rangle_A + \frac{(\gamma_A-\delta_A)}{\sqrt{2}}|-\rangle_A$, and $\frac{(\gamma_B+\delta_B)}{\sqrt{2}}|+\rangle_B + \frac{(\gamma_B-\delta_B)}{\sqrt{2}}|-\rangle_B$, where $\{|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle), |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)\}$ is the diagonal basis. Accordingly, the joint state on Photon A and Photon B is $\frac{(\gamma_A+\delta_A)(\gamma_B+\delta_B)}{2}|+\rangle_A|+\rangle_B + \frac{(\gamma_A+\delta_A)(\gamma_B-\delta_B)}{2}|+\rangle_A|-\rangle_B + \frac{(\gamma_A-\delta_A)(\gamma_B+\delta_B)}{2}|-\rangle_A|+\rangle_B + \frac{(\gamma_A-\delta_A)(\gamma_B-\delta_B)}{2}|-\rangle_A|-\rangle_B$. After the BS, the anti-correlated components in this joint state is transformed to be

$$\begin{aligned} & \frac{(\gamma_A+\delta_A)(\gamma_B-\delta_B)}{2}|+\rangle_A|-\rangle_B + \frac{(\gamma_A-\delta_A)(\gamma_B+\delta_B)}{2}|-\rangle_A|+\rangle_B \\ \rightarrow & \frac{(\delta_A\gamma_B-\gamma_A\delta_B)}{\sqrt{2}}|\Psi_{AB}^-\rangle_{\text{diagonal}} + \frac{i(\gamma_A\gamma_B-\delta_A\delta_B)}{\sqrt{2}}|\Psi_{AB}^+\rangle_{\text{diagonal}}. \end{aligned} \quad (10)$$

This relation is founded because that

$$\begin{aligned} |+-\rangle_{AB} & \rightarrow \frac{1}{\sqrt{2}}(i|\Psi_{AB}^+\rangle + |\Psi_{AB}^-\rangle)_{\text{diagonal}}, \\ |-+\rangle_{AB} & \rightarrow \frac{1}{\sqrt{2}}(i|\Psi_{AB}^+\rangle - |\Psi_{AB}^-\rangle)_{\text{diagonal}}. \end{aligned} \quad (11)$$

Here the subscript diagonal means these states are characterized in the diagonal basis. When the measurement basis is transformed from the diagonal basis to the rectilinear basis, $|\Psi_{AB}^-\rangle_{\text{diagonal}}$ is formally invariant, while $|\Psi_{AB}^+\rangle_{\text{diagonal}}$ is converted to be $|\Phi_{AB}^-\rangle$. Thus the probabilities of $|\Psi_{AB}^-\rangle_{\text{diagonal}}$ and $|\Psi_{AB}^+\rangle_{\text{diagonal}}$ may be different when Alice's and Bob's state are directly feeded in the Bell basis measurement device, and their total probability is calculated to be $\frac{(\gamma_A\gamma_B-\delta_A\delta_B)^2+(\gamma_A\delta_B-\delta_A\gamma_B)^2}{2}$.

When the bases of the PBSes are chosen to be aligned with the diagonal basis, the joint state on the photons emitting from the PBSes is $\frac{(\gamma_A+\delta_A)^2(\gamma_B+\delta_B)^2}{4}|++\rangle_{AB} + \frac{(\gamma_A+\delta_A)^2(\gamma_B-\delta_B)^2}{4}|+-\rangle_{AB} + \frac{(\gamma_A-\delta_A)^2(\gamma_B+\delta_B)^2}{4}|-+\rangle_{AB} + \frac{(\gamma_A-\delta_A)^2(\gamma_B-\delta_B)^2}{4}|--\rangle_{AB}$. The anti-correlated components of the state is converted to $|\Psi_{AB}^+\rangle_{\text{diagonal}}$ and $|\Psi_{AB}^-\rangle_{\text{diagonal}}$ in the Bell basis measurement. According to (11), both $|\Psi_{AB}^+\rangle_{\text{diagonal}}$ and $|\Psi_{AB}^-\rangle_{\text{diagonal}}$ happen with probability $\frac{(\gamma_A\gamma_B-\delta_A\delta_B)^2+(\gamma_A\delta_B-\delta_A\gamma_B)^2}{4}$. Thus their total probability is $\frac{(\gamma_A\gamma_B-\delta_A\delta_B)^2+(\gamma_A\delta_B-\delta_A\gamma_B)^2}{2}$, which is equal to that when no PBS is set in Alice's and Bob's labs. It means the PBSes in the diagonal basis can also pick out the anti-correlated events to be measured as $|\Psi_{AB}^+\rangle_{\text{diagonal}}$ and $|\Psi_{AB}^-\rangle_{\text{diagonal}}$ in the Bell basis measurement.

Experimentally, Alice's and Bob's bit value choices in every basis are randomly chosen. Namely, $\{\gamma_A|0\rangle_A + \delta_A|1\rangle_A, -\delta_A|0\rangle_A + \gamma_A|1\rangle_A\}$ are equiprobably prepared by Alice, and $\{\gamma_B|0\rangle_B + \delta_B|1\rangle_B, -\delta_B|0\rangle_B + \gamma_B|1\rangle_B\}$ are equiprobably prepared by Bob. Thus Both Alice's and Bob's states are characterized as identity matrix, and their joint state is an identity matrix in the 4-dimensional Hilbert space.

A modified time-reversal Bell test with the lhv theory excluded can be realized with the following steps.

(A) Alice has two copies of PD_A (PD_A¹, and PD_A²), and Bob has two copies of PD_B (PD_B¹ and PD_B²). Alice's bases are randomly chosen to be $A_1 = \sin\theta\hat{\sigma}_x + \cos\theta\hat{\sigma}_z$, $A_2 = \sin(\theta + \frac{\pi}{2})\hat{\sigma}_x + \cos(\theta + \frac{\pi}{2})\hat{\sigma}_z$. Bob's bases are randomly chosen to be $B_1 = \sin(\theta + \frac{\pi}{4})\hat{\sigma}_x + \cos(\theta + \frac{\pi}{4})\hat{\sigma}_z$, $B_2 = \sin(\theta - \frac{\pi}{4})\hat{\sigma}_x + \cos(\theta - \frac{\pi}{4})\hat{\sigma}_z$. Their bit values are randomly chosen as -1 and 1.

(B) According to their basis choices and bit value choices, Alice and Bob prepare their states in the labs. Every state prepared by Alice has two copies. One is for PD_A¹ and the other is for PD_A². Similarly, every state prepared by Bob also has two copies. One is for PD_B¹ and the other is for PD_B². The bases of the PBSes in PD_A¹ and PD_B¹ are aligned with the rectilinear basis, while the those of the PBSes in PD_A² and PD_B² are aligned with the diagonal basis.

(C) The photons with the well prepared states are fed into the PBSes. Alice sets her single-photon-detectors (SPDs) at every output of her PBSes, and so does Bob. They record all the registers on their detectors.

(D) Alice and Bob announce their measurement outcomes through the public channels. The events are kept when both PD_A¹ and PD_B¹ emit anti-correlated clicks, and simultaneously, both PD_A² and PD_B² also generate anti-correlated detections.

(E) The experiment goes on until Alice and Bob keep enough experimental data. They use their bit values and basis choices of the kept events to calculate the value of S_{CHSH} .

In step (C), Alice and Bob measure on the photons emitted out from their PBSes rather than forward them to Charlie. If the time intervals between Alice's and Bob's measurements are very short, the lhv at Alice's side cannot reach Bob's lab before Bob's photon is clicked on his detectors, and vice versa. When the Bell violation is obtained, from Eq.(6), it cannot be contributed by the lhvs. The anti-correlated results from PD_A¹ and PD_B¹ means that the measurement outcomes should be $|\Psi_{AB}^+\rangle$ or $|\Psi_{AB}^-\rangle$ if Photon A and Photon B passed the BS and were measured in the Bell basis. When Alice and Bob have anti-correlated results in PD_A² and PD_B², however, Charlie's measurement outcome should be $|\Phi_{AB}^-\rangle$ or $|\Psi_{AB}^-\rangle$ if the photons were not measured directly. Thus when the conditions in step (D) are satisfied, the measurement outcome of Charlie should be $|\Psi_{AB}^-\rangle$ if the photons at both sides passed the BS and were measured in the Bell basis.

For PD_A^1 and PD_B^1 , the state on their anti-correlated outputs is $\rho_{\text{anti-correlated}}^{\text{rectilinear}} = \frac{1}{2}(|01\rangle_{AB}\langle 01| + |10\rangle_{AB}\langle 10|)$. For PD_A^2 and PD_B^2 , however, the state on the anti-correlated outputs is $\rho_{\text{anti-correlated}}^{\text{diagonal}} = \frac{1}{2}(|+-\rangle_{AB}\langle +-| + |-+\rangle_{AB}\langle -+|)$. It is easy to verify that $\text{Tr}(\rho_{\text{anti-correlated}}^{\text{rectilinear}} \hat{\sigma}_z^A \otimes \hat{\sigma}_z^B) = -1$, $\text{Tr}(\rho_{\text{anti-correlated}}^{\text{diagonal}} \hat{\sigma}_x^A \otimes \hat{\sigma}_x^B) = -1$, and $\text{Tr}(\rho_{\text{anti-correlated}}^s \hat{\sigma}_z^A \otimes \hat{\sigma}_x^B) = \text{Tr}(\rho_{\text{anti-correlated}}^s \hat{\sigma}_x^A \otimes \hat{\sigma}_z^B) = 0$, with $s \in \{\text{rectilinear}, \text{diagonal}\}$. If there is a single state, $\rho_{\text{anti-correlated}}$, who combines the characters of both $\rho_{\text{anti-correlated}}^{\text{rectilinear}}$ and $\rho_{\text{anti-correlated}}^{\text{diagonal}}$, it can be proven to be $\rho_{\text{anti-correlated}} = |\Psi_{AB}^-\rangle\langle \Psi_{AB}^-|$, and $S_{\text{CHSH}}^{|\Psi_{AB}^- \rangle} = -2\sqrt{2}$. It means that if the conditions of step (D) in the modified time-reversal Bell test are satisfied, Alice and Bob succeed in selecting $S_{\text{CHSH}}^{|\Psi_{AB}^- \rangle}$. From the aspect of time-reversal formation of entanglement, the CHSH inequality is violated and the lhv theory is excluded in the the modified time-reversal Bell test. From (8) and (10), the experiment succeeds with probability $\frac{(\gamma_A^2 \delta_B^2 - \delta_A^2 \gamma_B^2)^2}{4}$.

V. DISCUSSION AND CONCLUSION

In this letter, we have raised the time-reversal Bell test protocol. Accordingly, we have proposed the lhv model for it. Analyses were given on the unfair-sampling lhv and the deterministic lhv which may contribute the Bell violation in the lhv theory. By simply measuring their own prepared states directly in the rectilinear basis or the diagonal basis, Alice and Bob are able to differentiate the anti-correlated components from the correlated components of the states destructively. In the rectilinear basis, the anti-correlated components contribute to $|\Psi_{AB}^+\rangle$ and $|\Psi_{AB}^-\rangle$. In the diagonal basis, the anti-correlated contribute to $|\Phi_{AB}^-\rangle$ and $|\Psi_{AB}^-\rangle$. In every run, Alice and Bob each prepares two copies of their states. The anti-correlated components of one copy of their joint state are distinguished in the rectilinear basis, and those of the other copy are distinguished in the diagonal basis. The overlapped anti-correlated results in their detectors should contribute to $|\Psi_{AB}^-\rangle$ if they passed the BS and were measured in the Bell basis. As Alice and Bob detect their states in their own labs, the lhvs are prevented at both sides to contact with each other. Thus the violation of the CHSH inequality supports the correctness of quantum principles and excludes the lhv theory. Our protocol only picks up the efficient detections of Alice and Bob locally, it does not require high detection efficiency. Thus it can be realized with present technology.

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Author contribution statement

Y.T wrote the manuscript text, drew Fig. 1, and reviewed the manuscript.

Competing financial interests

The authors declare no competing financial interests.